



**TITLE:**

**To use a collaborative teaching approach across a group of small Primary schools with the aim of improving proficiency and fluency in multiplication and division.**

**Ref No – CTP0813**

**Abstract:**

This project was designed to address the disparity in the teaching of multiplication and division to Y2/3 children across a group of small primary schools. The aim was to improve the learning of multiplication and division as these skills are first introduced so as to ensure the children develop a deeper understanding of these arithmetical concepts.

Smaller primary schools are often without a specialist maths teacher. An effective program of collaborative teaching between schools should provide valuable support for primary colleagues for whom mathematics is not their specialist subject. The logistics of implementing a CTP across a group of schools is challenging; our experience is that it is most effective for schools which share a common objective. We decided to use the recently formed federation groupings rather than geographical proximity.

Each federation planned their own strategy for the teaching of multiplication and division at Y2/3. The implementation of the strategy was observed in lessons over a term and findings were discussed in meetings to assess the specific learning outcomes.

The project has seen an increased degree of consistency in the teaching of multiplication and division and an improvement in teaching strategies within the schools involved. Improved teacher confidence in introducing these concepts has been recorded across both federations. Assessments have shown that the children directly involved in the project have made progress in their understanding of both multiplication and division although division is still a challenging concept.

**Aims:**

The focus of our project was to develop strategies to improve children's understanding and use of multiplication and division from KS1 through to KS2 and, in particular, the project aimed to respond to the identified need at two of our local Primary schools to improve mathematical attainment at KS1.

As the destination secondary school, Kingsbridge Community College had noted an inconsistency in arithmetical skills and methods used by year 7 students upon entry and this project provided an opportunity to begin to address this disparity. The real benefits of the project were expected to be

twofold - directly impacting the learning of the children involved in the project and indirectly by promoting a more consistent approach and improved strategies for the teaching of these skills.

Furthermore, by focusing on the concepts of multiplication and division at this early stage, it is our aim that these children gain a firmer foundation in these skills which will lead to improved fluency and proficiency in later years.

A secondary equally important objective was to 'kick start' a collaborative approach across the groups of our smaller feeder Primaries to facilitate an on-going program to improve the teaching and learning of mathematics in general. In our experience the recommendation of The Williams report (2011) that 'within ten years there should be at least one Mathematics Specialist in every primary school' is an ideal which has yet to be implemented in many smaller Primaries. One way to tackle this lack of expertise is to adopt collaborative practice between schools so that the skills and knowledge of a SST may be more readily shared. Research indicates that collaborative teaching offers immediate impact in the classroom and provides a system of on-going support for primary colleagues for whom mathematics is not their specialist subject. Our aim is to instigate enthusiastic and confident teaching of mathematics and to gain improved strategies for teaching multiplication and division which are consistent across the group of schools.

### **How was our project organised?**

In order to make the logistics of teacher collaboration between different schools as straightforward as possible the groupings for the collaborative practice were organised separately within two existing federations. Each of the federations included schools of small to medium numbers on roll. Both federations currently employ one maths specialist teacher working at one of their schools.

- 'Kingsbridge Area Church School Federation':

All Saints' Primary, Thurlestone      West Charleton Primary      West Alvington Primary

These three schools participated in the form of two triads of teachers: three year 2 teachers and three year 3 teachers.

Ruth Benton is the maths co-ordinator for this federation; she is also a maths specialist and SLE for the Kingsbridge area.

- 'Our School Federation':

East Allington Primary      Stoke Fleming Primary      Blackawton Primary

Seven teachers participated organised into Y2&3 teachers working together within each school. Claire Widdicombe, the maths leader for this federation, also joined the planning sessions for each group.

The role of the Primary Maths Co-ordinator at Kingsbridge Community College was held by Jacky Walker who was the project organiser and was available to provide maths support.

Over a number of years the destination secondary school, Kingsbridge Community College, had noted a steady decline in the standards of numeracy for the year 7 students on entry to the college. During the previous academic year (2011/12) KCC and the Kingsbridge Area Schools Partnership (KASP) had implemented a program of action involving ten feeder Primary schools designed to improve standards of numeracy. One strand of this program had been to pilot a lesson study approach in spring 2012 with the aim of improving skills for individual teachers in the feeder primaries who had been identified as lacking confidence in their mathematics teaching. This program of joint practice development had proved to be particularly effective for our largest feeder primary where the logistics of organising collaborative teaching for parallel classes had been relatively straightforward. Indeed, this primary school has since adopted an on-going collaborative approach to CPD in maths. However, there had been more limited success adopting a truly collaborative approach across disparate small schools where there had been no history of collaboration. Our experience demonstrated that small schools which had been in 'competition' due to falling numbers on roll did not participate effectively in collaborative practice. The one notable exception to this occurred for a particular triad of teachers in schools which had recently joined together as a new federation. Despite these schools being geographically further apart this group of participating teachers were enthusiastic about the merits of joint practice development. Such a positive experience has encouraged us to join this NCETM collaborative teaching project in order to trial a program across groups of smaller feeder primaries within existing federation groupings.

### **What did the project involve?**

The project was launched at the start of the summer term with two planning meetings organised separately for the year groups involved (years 2 & 3) in which teachers discussed an overview of the proposed program of lesson study cycles and planned their first lesson. At this stage, the year 2 teachers decided to focus their three lessons as two on multiplication followed by one on division; the year 3 teachers were to implement only two cycles, the first focussed on multiplication followed by a second on division.

Each 'lesson study' cycle involved 4 steps:

1. Teachers of parallel classes meet to share ideas and plan a lesson for a pre-determined objective.
2. One teacher delivers the lesson to their class; the other teachers observe the learning of the children.
3. Teachers meet together to review the observed lesson and revise their original plan.
4. Teachers use the revised plan for a lesson with their own individual class.

Although both federations worked together at the initial launch meetings, it is interesting to note that the teacher triads from each federation followed quite different strategies in their respective lesson study cycles.

### The 'Kingsbridge Area Church School Federation' Strategy:

This federation comprises schools which are comparatively close together geographically and the group followed closely the above model of collaboration between parallel classes. The triads from this federation developed lesson plans for both multiplication and division based on the idea of an 'array hunt' as described in the publication 'Maths Outside the Classroom' by the Devon Maths Team.

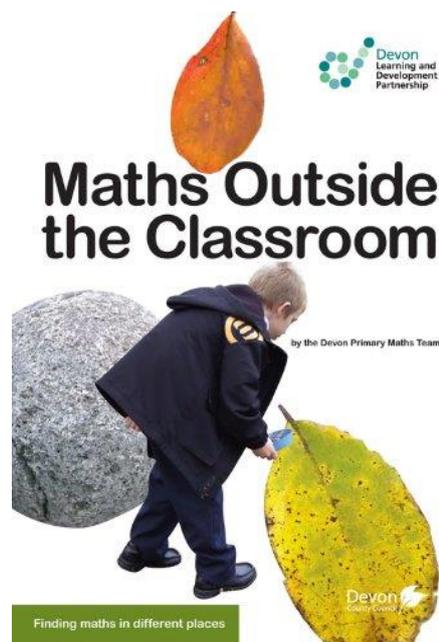
The year 2 teachers' initial approach was to instigate an investigation whereby the children worked in a group to collect their own large sets of items from outside the classroom (e.g. pebbles or daisies). The children were then asked to find an efficient way to count their sets accurately.

The teachers reported two clear learning benefits of this strategy which were quickly identified: Firstly, the children displayed enthusiasm and a desire to find out how many they had collected; secondly, the strategy saw children follow a natural progression to the use of multiples as an efficient and accurate method in which to count their objects. The children quickly learned that organising their objects into an array and using multiples for counting was easier and consistently more accurate than trying to count the objects in a long line. Furthermore, the children's desire to show their peers the items they had collected appeared to give the children greater confidence as they discussed their methods for counting.

It is interesting to note that this task produced different arrays dependant on the size and number of the objects collected and the more able children did not necessarily choose 'easy' multiples such as 2, 5 or 10.

Furthermore, the fact that the children constructed their own arrays enabled them to more intuitively understand the need to order the objects directly in line, one above the other, so that they had exactly the same number in each row. In the first instance some children organised their items into rows which did not all contain the same number of items (usually because they had chosen items of different size) but, after sharing their ideas, most children adapted their own arrays themselves so that they had the same number in each row. During subsequent lessons they developed the idea of using an array with 'remainders' in a pot to add on afterwards as a useful adaptation.

The year 3 triad of teachers also planned an investigational activity using arrays although they opted for a more structured approach to encourage confident use of the multiplication sign. The initial images of egg boxes and chocolate bars provided a quick introduction to the concept of an array and the materials used for their investigations involved everyday objects (such as Cheerios cereal) alongside classroom apparatus. Although the initial approach was to use an array as a quick way to count a pile of Cheerios accurately, the year 3 triad decided to structure the lesson more rigidly after this by giving the children a precise number of objects which they were asked to present in a specific



array. The step between using an array as a counting aid to using an array to demonstrate a multiplication calculation was found to be a natural progression.

The group also reported some bonus concepts arising from their further lessons using arrays: for example, the use of arrays proved particularly beneficial to the children's progression to the understanding of the commutativity of multiplication such as  $3 \times 4 = 4 \times 3$ . Further investigations which required the children to order 36 Cheerios in different ways also gave rise to the bonus concept that a 'square array' indicates that 36 is a 'square number'.

Both the year 2 and year 3 triads continued to use their array strategy for their lesson on division. To avoid the added complication of remainders at this stage, the number of objects and array dimensions was specified by the teacher. The strategy continued to be practical so that the children were physically involved in arranging the objects into arrays in order to answer a division problem. The year 2 children were mainly thinking about a real problem such as 'how many Cheerios in each row?'; some of the year 3 children were able to use their arrays as a practical image to demonstrate more abstract division such as  $12 \div 3$ .

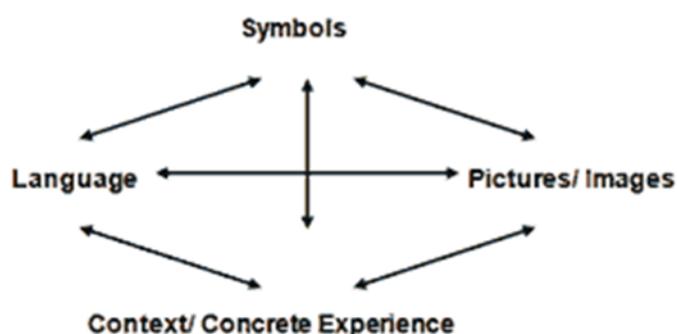
### **The 'Our School Federation' Strategy:**

Due to the geographical challenges of travelling long distances between schools for Our School Federation, it was decided to adapt the lesson study model so that the year 2/3 teachers worked together in each school with their maths co-ordinator providing specialist support as appropriate. This modification also allowed a more efficient distribution of funding which enabled the year 3 teachers to participate in three cycles of lesson study. To compensate for this loss of direct collaboration between schools, a series of planning and feedback sessions at federation maths team meetings were planned right from the start in order to share findings and to maximise the impact of the project across all of the schools in this federation.

Although the three schools followed individual lesson study plans they had decided on a consistent strategy to launch their cycle on multiplication and division:

Using the connective model of teaching mathematics, developed by Derek Haylock and Anne Cockburn (Haylock and Cockburn; 1989), teachers explored how experiencing and connecting these specific mathematical elements could help pupils to develop their multiplication and division skills.

The Connective Model



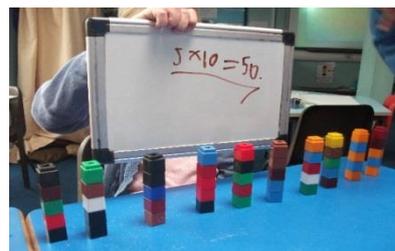
The children were organised into groups and given a 'real life word problem' to solve which was designed so as to use repeated addition or multiplication.

For example:

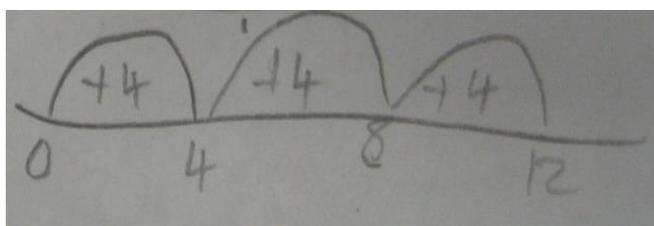
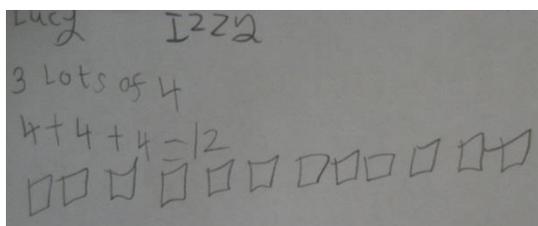
'Apples come in bags of 4.

Amy bought 3 bags.

How many apples does she have?'



General classroom apparatus was available for use. The groups were encouraged to record their findings in as many different ways as possible. One focus for this first lesson on multiplication was to present solutions to problems in an organised way; explain decisions, methods and results in pictorial, spoken or written form, using mathematical language and number sentences.



The second cycle lesson on multiplication was more varied for each school in response to the feedback from the first cycle. It is interesting to note that the East Allington teachers decided to change direction away from using array diagrams after their first cycle in order to follow through the methods used by their children who had preferred using the number line.

At Blackawton it was found that children grasped the word problem in different ways. Whilst the main strategy which children used was to count out the objects into equal groups, others quickly learned to use multiplication as their strategy.

'You can just count in twos until you've counted it three times'

When asked if  $2 \times 3$  is the same, the child answered:

'It is a different sum but it has the same answer.'



At Stoke Fleming the children were organised into differentiated maths groups. In response to their 'word problem' the children displayed all of the following strategies:

- Making arrays
- Drawing images
- Using equipment – beadstrings, numicon, unifix cubes, pencils
- Writing number sentences – multiplication and repeated addition
- Writing out times tables

The pupils used different strategies according to their ability – the most able pupils relied on some images, but mainly number sentences and their knowledge of the times tables; others used a range

of equipment and simple images and were able to talk about what they were doing, with some support; the least able group initially drew images, but then used equipment to solve the problem. This triad reported that mathematical language must be introduced when discussing the word problem in order for the learning to be more focused on multiplication.

The strategy for the following lesson was to present the children with a number sentence (such as  $3 \times 4$ ,  $6 \times 4$ ,  $9 \times 4$ ) in middle of large sheet of paper which they were asked to represent with a word problem, equipment, repeated number sentence and images. Pupils were reminded of different representations of a multiplication number sentence:

- word problem
- number sentence using 'x'
- pictures and images
- arrays
- sharing and division
- repeated addition
- equipment

They chose a multiplication sentence to represent in a variety of forms. All pupils settled down quickly and were fully engaged in this activity.



The most able pupils left using equipment to the last! They chose a large number sentence  $19 \times 11$  which they later realised was 'too big' when it came to representing the number sentence with apparatus. These pupils successfully wrote a word problem and were able to show a related division calculation. The other groups all represented their number sentence with equipment and arrays but found that making their own word problems was challenging. All groups appeared more confident, tackling the task with more independence and assurance. They used mathematical language well and chose appropriate number sentences, many choosing to count in multiples of 5 or 10.

These activities were repeated in Y2 with different outcomes. The 'word problem' and 'number sentence' were presented using known times table facts and the children effectively used equipment and arrays, both very visual and tactile representations, but they found the more formal aspects very challenging: they still saw multiplication as 'lots of' and 'groups of,' not as times or multiply.

### **Project Schedule:**

The pre-cycle assessments of oral and written skills in multiplication & division were carried out on schedule during week 1 of the project.

The schedule of collaborative teaching was closely followed throughout the project. The time period for each cycle of lesson study required approximately 2 weeks to allow for all teachers to observe lessons, provide feedback and then reassess the plan. Although it was possible to combine the review of one lesson with the planning of the next, each cycle was found to require 2 weeks as planned.

With the post cycle assessments the schedule over ran due to special events affecting the regular school lessons at the end of the summer term. This affected the overall project schedule by prolonging the time period. For this reason it was decided that it would be more efficient to combine the post-cycle assessments of multiplication and division. These assessments were carried out in the final week of term.

### **What has been learned?**

In terms of the teaching strategies trialled in this project, the following has been learned:

The 'Kingsbridge Area Church School Federation' found many benefits of their array strategy from the natural progression of using arrays to count in multiples to using arrays to support understanding of multiplication and further to the linking of the concept of division. The introduction of division using arrays was found to be better if the number of items was specified by the teacher so as to avoid remainders at this stage. It was also evident that the action of collecting their own items enabled the children to take ownership of their array method which encouraged a deeper understanding of the concept of multiplication. Possibly the most important outcome, however, is that children were enthusiastic and confident as they discussed their calculation methods.

The research of the 'Our School Federation' triads suggests that the learning of multiplication and division should reflect the individual needs of the child. Although arrays formed part of the learning strategies trialled in both year 2 and year 3, they found that some children responded better to alternative strategies.

Through this project teachers were able to observe, and then discuss, the thinking of the pupils as they tackled a range of problems. The 'Our School Federation' reported that misconceptions and approaches to these problems were many and varied across the classroom and unsurprisingly that more able children relied less on apparatus, pictures and images to answer a problem, but instead used symbols and learned calculation strategies to help them. However, it was noted that when challenged with a problem involving division, even the more able pupils reverted to using classroom apparatus and images.

Carrying out this research with both Years 2 and 3 gave this group of schools the opportunity to really think about the age at which the majority of children are ready to learn about multiplication and division. They discovered that most Y3 children are ready for the challenges of formal multiplication; that they have a wider use of mathematical language and have a wider use and

knowledge of multiplication strategies. They are less confident when using division. Whereas Y2 children were still working with 'lots of' and 'groups of'. The use of mathematical language by Y2 children is limited and that the majority require visual, tactile and practical experiences to consolidate understanding.

In terms of the logistics of the implementation of collaborative practice across a group of smaller primary schools, the project has demonstrated that it is effective where schools share a common objective and that this may be a more important factor than geographical proximity, within reason. Our experience suggests that collaboration is particularly beneficial for groups of teachers working at different schools within an organised federation. Both participating federations in this project have reported benefits for individual teachers involved and a wider impact on the schools as a whole. They have learned that a collaborative approach can be effective at tackling individual teacher weaknesses in the teaching of mathematics and can be the means to research improved teaching strategies which are immediately shared across parallel classes resulting in a more consistent standard of teaching and learning across the group of different schools.

### **What has been the impact of this project?**

It is important to note that maths is not the most confident area of the curriculum for several teachers involved in this project and, on this level alone, this project has had direct impact on individual teacher practice by providing an improved strategy for the teaching of multiplication and division. For example, prior to the project some of the participating teachers said that they had never used arrays in their classrooms to demonstrate multiplication. All of the teachers involved in this project are enthusiastic about the merits of joint practice development; they reported a satisfaction in collaboratively planning a new strategy which they were then able to use immediately in their own classroom.

This collaborative project has also provided maths leaders the opportunity to work alongside teachers in their schools to identify strengths in maths teaching and to provide constructive support where needed in a non-threatening way (for example, the maths leader being the one in the triad to do the teaching on some occasions to model practice instead of simply observing or monitoring).

The maths leader at the Our School federation has reported evidence of teachers adapting planning to ensure that meaningful links are now being made between the different areas of maths (e.g. fractions and division) within their teaching.

Assessment data (from both tests and teacher observations) has shown an improvement in children's proficiency and fluency in multiplication and division over the term of the project. The maths leaders report that the children in the participating classes are evidently more comfortable in using these concepts. A more detailed analysis of the results of the assessments have shown that the pupils have gained in confidence when calculating multiplication and division problems in a range of ways although it appears that division remains a more challenging concept.

The project's findings have been shared in maths leaders' meetings for both federations where the discussions have been positive, constructive and led to an improvement in teaching and learning in mathematics. For the Kingsbridge area federation the array strategy has been embedded into their scheme of work; for the Our School federation the findings have been shared with teachers of Y1 to Y6 in all of their schools. Medium Term planning for Autumn Term 2013 has been adapted in light of findings from the project.

As a direct result of the strategies used in this project a calculation policy for teaching multiplication and division (as grouping) word problems was designed by the Y2/3 teachers in East Allington, Blackawton and Stoke Fleming. The use of the number line and the connective model is now at the heart of teaching multiplication/division across the Our School Federation. As the children helped to devise the calculation policy, teachers are hopeful that it will therefore be more meaningful and relevant for them. By devising the policy and ensuring teachers are more aware of how children progress, there will be a greater emphasis on children moving through the different stages of development more quickly e.g. moving older, more able pupils onto short, efficient methods for calculation. This policy has been shared with the teachers in Y5/6 with a view to extension and this draft programme of study for teaching multiplication and division is attached to this report.

The aim of the calculation policy is that children will have a secure understanding of multiplication and division and understand the relationship between the two concepts before they leave primary school.

#### **Advice to teachers:**

Although the logistics of organising collaborative teaching between schools may appear daunting, the benefits are evident for both teachers and children. The participating teachers have gained a more confident and enthusiastic attitude to the teaching of the concepts covered in the program and the schools have gained improved strategies which are consistent across the group. It is imperative to consider most carefully the groupings of schools for a collaborative teaching program. Our experience shows that collaborative teaching is most effective if schools share a common objective such as being member schools of the same federation.

We have also found a practical advantage to organising a series of lesson study cycles to run sequentially; this enables step 3 (feedback & re-planning) of one cycle to be combined with step 1 (planning) of the next. It is also possible to minimise travel between schools by blocking the time after the observed lesson (step 2) for the teachers to feedback and plan. Hence it was found that, after the initial planning meeting for the triad of collaborating teachers, the most efficient method to implement a series of collaborative teaching cycles is to organise the lesson observation (step 2) to be followed directly by the feedback / re-planning of that lesson (step 3) and the collaborative planning for the next cycle (step 1).

For schools which are geographically isolated, the adaptation followed by the Our School Federation (where teachers from adjacent year groups in individual schools collaborated together in order to minimise travel/time costs) is effective so long as a maths specialist or a confident maths teacher is routinely involved in at least the planning stage. In this situation it is also imperative to plan opportunities for regular dissemination of what they have learned at meetings of teachers of parallel classes at other schools within the group/federation.

As an extension to this model, the Our School Federation has also trialled team-teaching of lessons which have been collaboratively planned. (This involved 2 teachers teaching and one teacher observing). They reported that team teaching can be most effective in enabling teachers to focus on the learning and therefore ensure that assessment and observations are at the heart of the process.

**References:**

NRich Articles on Arrays, Multiplication and Division. (1997 – 2012) <http://nrich.maths.org/8773>  
The Connective Model of Teaching Mathematics. (Haylock and Cockburn; 1989),  
Maths Outside the Classroom – Finding Maths in Different Places (Devon Learning and Development Partnership. Devon LDP Primary Maths Team; 2011)

**Resource produced:**

A draft proposal of study for the teaching of multiplication and division from Y2 – Y6. This document can be found attached to this report.

**Details of dissemination:**

The main focus for dissemination has been local to Kingsbridge with a view to starting further collaborative teaching programs in maths. Following our report of this successful project another local primary has included collaborative teaching in their plan for maths this academic year; Ruth Benton will be working with this school sharing her experience of this project.

Further dissemination will be through the Kingsbridge Area Schools Partnership meetings of maths co-ordinators. In January this meeting to be held at KCC will focus on the specific strategies for the teaching of multiplication and division trialled in this project and will discuss the collaborative approach.

This report will be made available to all primary schools in the South West Teaching School Alliance who will be invited to attend a conference to publicise the outcomes of the project. We will also share these with neighbouring Teaching Schools through the SW Teaching School Forum.